

# Construction of a multi-UAV 3-D path planning model

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**Abstract:** With the rapid development of science and technology, the key role of UAV in the military and other fields is becoming increasingly prominent, and multi-UAV path planning has also become the core element of the mission planning system. This paper focuses on the problem of multi-UAV three-dimensional path planning, establishes the three-dimensional spatial environment model, uses the digital elevation model modeling, and uses the statistical results of the evaluation function, aiming to solve the problems of slow search speed and poor quality of planning path during multi-threat three-dimensional spatial path planning, and puts forward effective multi-UAV flight path.

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## 1 UAV 3-D spatial path model construction

### 1.1 Terrain modeling

Accurate terrain modeling is the foundation of UAV and the critical link. This paper uses the digital elevation model (Digital Elevation Model, DEM) to realize the precise description of the terrain. DEM can directly reflect the fluctuations of the ground, provide accurate terrain information for UAV path planning, and ensure that the UAV can reasonably avoid terrain obstacles such as mountains and hills during flight.

#### 1.1.1 Digital model

The original digital terrain model simulates the height change of the real terrain through a complex mathematical expression:

$$h_1(x, y) = \sin(A + y) + B \cdot \sin x + C \cdot \cos(D \cdot \sqrt{x^2 + y^2}) + E \cdot \cos y + F \cdot \sin(G \cdot \sqrt{x^2 + y^2}) \quad (1)$$

Where,  $x, y$  are the projection coordinates of the horizontal plane,  $h_1(x, y)$  represents the height of the projection coordinates of the corresponding horizontal plane;  $A, B, C, D, E, F, G$  are the coefficient and change the terrain by adjusting the coefficient value<sup>[1]</sup>.

#### 1.1.2 Threat terrain model

In order to accurately reflect various threats in the model, an equivalent threat terrain model is constructed:

$$h_2(x, y) = \sum_{i=0}^k h(i) \cdot \exp \left[ - \left( \frac{x-x_{oi}}{x_{si}} \right)^2 - \left( \frac{y-y_{oi}}{y_{si}} \right)^2 \right] \quad (2)$$

Where:  $x, y$  are the projection coordinates of the horizontal plane,  $h_2(x, y)$  is the peak height corresponding to  $x$  and  $y$ ;  $h(i)$  Represents the highest point height of the peak  $i$ ;  $x_{oi}, y_{oi}$  is the horizontal plane projection coordinates corresponding to the highest point of peak  $i$ ;  $x_{si}, y_{si}$  Variables related to the slopes of peaks  $i$  along the  $x$ - and  $y$ -axes, When the height is determined,  $x_{si}, y_{si}$  The smaller the mountain, the steeper the mountain<sup>[1]</sup>.

#### 1.1.3 The 3 D spatial model

Combine the original digital terrain model with the equivalent threat terrain model to construct the basic 3 D spatial model:

$$h(x, y) = \max(h_1(x, y), h_2(x, y)) \quad (3)$$

This model considers terrain and threat factors, taking the maximum value of both in the same coordinate position as the effective height of the point. In a coordinate position, if the height of the original terrain is higher than the threat equivalent height, the original terrain height shall prevail, indicating that the position is mainly affected by the terrain; otherwise, if the threat equivalent height is higher than the original terrain height, the threat equivalent height shall prevail, indicating that the threat in this position has a greater impact on the flight of the UAV. In this way, it can effectively avoid the threat areas and ensure the flight safety of UAV.

### 1.2 Threat modeling

In the flight process of UAV, we are faced with many threat factors, which seriously affect the flight safety of UAV and the success rate of mission execution. The enemy air defense system is one of the most important threats, equipped with its advanced radar capable of accurately detecting the UAV, and once the UAV is found, it will quickly guide the anti-aircraft missiles to attack. Enemy anti-aircraft guns also have a high rate of fire and a large fire coverage, posing a great threat to low-flying UAV.

To accurately consider these threats in pathway planning, the threat sources are modeled in detail. Each threat source is treated as an object with specific properties, including location, threat radius, etc. The location of the threat source is determined by its coordinates in three-dimensional space  $x_o, y_o, z_o$ , which defines the specific location of the threat source in space. The threat radius (R) represents the range of influence of the threat source, within which the UAV faces a high risk. When the UAV enters within the threat radius of the threat source, its probability of being attacked or damaged increases significantly.

### 1.3 Evaluation function

The evaluation function constructed in this paper consists of four parts: path length, threat, height, and smoothing cost, to comprehensively evaluate the quality of path planning.

#### 1.3.1 Path-length cost

The path length cost reflects the energy cost and time cost required for UAV flight:

$$F_1(\text{path}_{ij}) = \sum_{k=1}^{n-1} \|L_{ij}^k L_{ij}^{k+1}\| \quad (4)$$

among,  $\|L_{ij}^k L_{ij}^{k+1}\|$  Euclidean distance between the k th path point and the k+1 path point in the UAV execution target  $T_i$ . n is the total number of path points where the UAV performs the target  $T_i$ . Shorter path length means that the drone consumes less fuel, shorter flight time and more efficient mission execution.

#### 1.3.2 Path threat cost

The path threat cost takes into account the various threats faced by drones during flight. Therefore, the planning path should not only avoid obstacles and threat sources but also leave a certain safety redundant distance. The calculation formula is as follows:

$$B_k(L_{ij}^k L_{ij}^{k+1}) = \begin{cases} 0, & D_k^{k+1} > R_m + Z + E \\ R_m + Z + E - D_k^{k+1}, & Z < D_k^{k+1} - R_m \leq Z + E \end{cases} \quad (5)$$

$$F_2(\text{path}_{ij}) = \sum_{k=1}^{n-1} \sum_{m=1}^M B_k(L_{ij}^k L_{ij}^{k+1}) \quad (6)$$

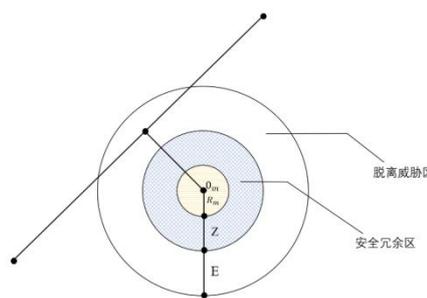


Fig 1. Schematic diagram of the path threat cost

among,  $B_k(L_{ij}^k L_{ij}^{k+1})$  It represents the threat cost between the k th path point and the k+1 path point. M is the total number of threat sources within the task area.

$D_k^{k+1}$ , is the distance from path  $\|L_{ij}^k L_{ij}^{k+1}\|$  to the threat source.  $R_m$  is the threat radius of the m- th threat source. Z is the mandatory safety - redundant distance. E is the threat distance<sup>[1]</sup>.

#### 1.3.3 Route height cost

The path altitude cost mainly considers the rationality of the UAV flight altitude:

$$F_3(\text{path}_{ij}) = \begin{cases} \left| H_{ij} - \frac{(H_{\max} + H_{\min})}{2} \right|, & H_{\min} < H_{ij} < H_{\max} \\ \infty, & \text{else} \end{cases} \quad (7)$$

Where indicated,  $(H_{\max})$  and  $(H_{\min})$  are the maximum and minimum heights, respectively. The rationality of the path height is evaluated by calculating the deviation of the path height from the central values of the reasonable height range.

### 1.3.4 Path smoothing cost

The path smoothness cost is used to measure the smoothness of the path, as the drone turns and climbs to increase the body control difficulty and oil consumption. The path smoothing cost includes steering cost and climbing cost, calculated as:

steering angle:

$$\theta_{ij} = \arctan\left(\frac{\|L_{ij}^k L_{ij}^{k+1} \times L_{ij}^{k+1} L_{ij}^{k+2}\|}{\|L_{ij}^{k+1} L_{ij}^{k+2}, L_{ij}^{k+1} L_{ij}^{k+2}\|}\right) \quad (8)$$

Climbing Angle:

$$\delta_{ij} = \arctan\left(\frac{z_{ij}^{k+1} - z_{ij}^k}{\|L_{ij}^k L_{ij}^{k+1}\|}\right) \quad (9)$$

Path smoothing cost:

$$F_4(\text{path}_{ij}) = \omega_1 \sum_{k=1}^{n-2} \theta_{ij}^k + \omega_2 \sum_{k=1}^{n-1} |\delta_{ij}^k - \delta_{ij}^{k-1}| \quad (10)$$

Where, the steering angle  $\theta_{ij}$  is the angle  $L_{ij}^k L_{ij}^{k+1}$  and  $L_{ij}^{k+1} L_{ij}^{k+2}$  of two continuous path segments projected in the horizontal plane,  $L_{ij}^{k+1} L_{ij}^{k+2}$  is the horizontal projection of the UAV path from the K+1 path to the K+2 path point;  $\delta_{ij}$  is the climbing angle of the UAV path segment. Z is the unit vector in the forward direction of the delay z-axis.  $\omega_1$  and  $\omega_2$  is the weight of the steering cost and the climbing cost, respectively. The smoother the path, the smaller the change in the steering

Considering the above four parts, the path planning evaluation function of each UAV is:

$$F_{\text{all}}(\text{path}_{ij}) = \lambda_1 \cdot F_1(\text{path}_{ij}) + \lambda_2 \cdot F_2(\text{path}_{ij}) + \lambda_3 \cdot F_3(\text{path}_{ij}) + \lambda_4 \cdot F_4(\text{path}_{ij}) \quad (11)$$

Where  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  are, respectively, the path length, the cost path threatens the cost, and the weight of the path height.

## 2 Application and solution of the model

### 2.1 Background Introduction:

This mountainous area is undulating, peaks, valleys crisscross, the terrain is extremely complex. Through high-precision digital elevation model (DEM) data, the topographic and landform features of this area are described in detail. The highest peak is 2000 meters, and the lowest valley is 500 meters, In addition, due to the unstable geological conditions, some areas have rock fall risk, which are included in the consideration of obstacles.

The logistics mission requires drones to ship a batch of emergency supplies from a distribution center on the edge of the mountain to several designated locations inside the mountain within a specified time. The coordinates of the distribution center are  $(x_0, y_0, z_0)$ , while each target location is distributed in different topographic locations, and its coordinates are respectively  $x_1 y_1 z_1, x_2 y_2 z_2, x_3 y_3 z_3$ , etc. The mission time limit is relatively strict, requiring the UAV to complete the distribution of all materials within 2 hours, which puts forward high requirements on the efficiency of path planning.

The drone also faces a series of constraints during its flight. For example, the maximum flight altitude of the UAV is limited to 1500 m to avoid entering no-fly areas or being affected by high airflow; the minimum flight altitude should be kept above 100 m to ensure the obstacle. At the same time, the flight speed of the UAV is also limited, the maximum flight speed of 50 m / s, the minimum flight speed of 10 m / s, to ensure the stability and safety of the flight.

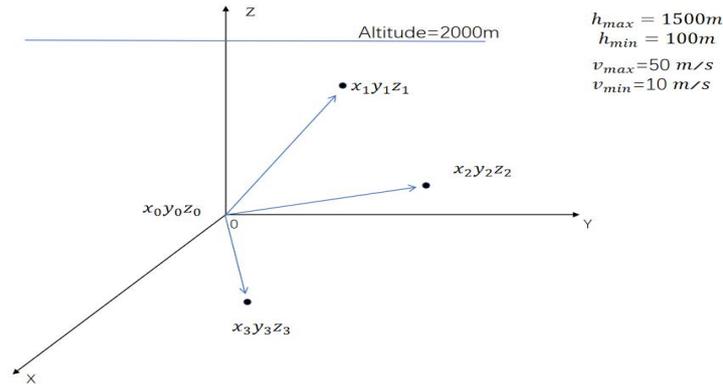


Fig 2. Background application model

## 2.2 Solution

### 2.2.1 Method based on the search algorithm

Such as the A algorithm, the Dijkstra algorithm, etc. These algorithms consider factors such as path length, obstacle, by searching the path in the graph or grid. For example, in simple two-dimensional or three-dimensional static environments, the A algorithm can quickly find a feasible path according to the heuristic function, but may be less efficient when dealing with high-dimensional space and complex constraints.

### 2.2.2 Group intelligence optimization algorithm

Particle swarm algorithm (PSO): regards the path of the UAV as the position of the particle in the search space, and find the optimal path by updating the speed and position of the particle.

Genetic algorithm (GA): to encode the drone path to simulate the genetic process of biological evolution.

Ant colony algorithm (ACO): Using the principle that ants release pheromone in the process of finding food, the UAV chooses the next flight direction according to the pheromone concentration and heuristic information on the path in the path planning.

### 2.2.3 Sampling algorithm

For example, the fast expansion random tree (RRT) algorithm randomly generates a series of nodes starting from the starting point, and continuously expands the tree structure to find a path connecting the starting point and the end point.

## 3 Summary and Outlook

### 3.1 Summary

This paper focuses on the construction of UAV 3-D path planning model and provides a comprehensive framework for UAV path planning through accurate modeling, setting constraints, and building evaluation functions. In terms of three-dimensional spatial environment modeling, the digital elevation model is used to achieve accurate terrain description. The original digital terrain model simulates terrain height changes with the help of complex mathematical expressions, and can present various terrain by adjusting the parameters. The equivalent threat terrain model superpositions multiple threat sources and can describe the location, intensity, and influence range of threat sources. The two are combined to form a comprehensive three-dimensional spatial model. At the same time, the threat source is modeled in detail, its location and threat radius is defined, and the UAV is guided to avoid threats by strategies such as setting a buffer zone during the path planning.

### 3.2 Outlook

Future research should aim to enhance the adaptability of path planning models to complex dynamic environments, sense environmental changes in real-time quickly adjust paths, and ensure the success rate of drone task execution in a variety of complex scenarios. Real-time sensor data can be introduced to realize the dynamic monitoring of the environment and online path optimization<sup>[3]</sup>.

## References

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